2016 John O'Bryan Mathematics Competition
5-Person Team Test

Abbreviated Instructions: Answer each of the following questions **using separate sheet(s) of paper for each numbered problem.** Place your team code in the upper right corner of each page that will be turned in (failure to do this will result in no score). Place problem numbers in the upper left corner. Problems are equally weighted; **teams must show complete solutions**

[redacted]

(not just answers) to receive credit. More specific instructions are read verbally at the beginning of the test.

1. If $x^2+x+1=0$, then

[redacted]

c. Find the value of $\left(x+\frac{1}{x}\right)^2+\left(x+\frac{1}{x}\right)^3+\left(x+\frac{1}{x}\right)^4+\left(x+\frac{1}{x}\right)^5$.

2. It has been your lifelong dream to paint this on your dorm room wall:

3. Two ferry boats start at the same instant from opposite sides of a river with parallel shorelines, traveling across the water on routes at rights angles to the shore. Each boat travels at a constant speed, although one is faster than the other. They pass at a point 720 yards from the nearest shore, and upon reaching the opposite shore, each boat docks for exactly 10 minutes. After docking for 10 minutes, each boat heads back to its original

Team Solutions

1.

a. The product is $x^3=1$. Then note that $x^2= 1/x$, $x=1/x^2$, $1= 1/x^3$

b. $(x+1/x)^2 = x^2+1/x^2+2 =(x^2+x+1)+1=1$

c. $(x+1/x)^3=(x+1/x)\ 1= x+x^2=(1+x+x^2)-1=-1$
$(x + 1/x)^4= ((x + 1/x)^2)^2=1^2=1$
$(x+1/x)^5= (x+1/x)^2\ (x+1/x)^3 = 1\ (-1) =-1$

So the sum must be 1+ (-1) + (1) + (-1) =0.

2.

Put in a coordinate system with the origin at the center of the middle large disk. Since it has radius 1, the points where the four other large disks are tangent to each other is a distance of 1 from the origin. This means the

coordinates for the large disks are (1,1) (-1,1) (-1,-1) (1,-1). Draw the

This means the diameter of a small circle is $\sqrt{2}+1-1=\sqrt{2}$ or the radius is $\frac{\sqrt{2}}{2}$.

1. The off-white paint needs to cover the area of the four small disks:

2. The medium light gray paint needs to cover the area of the 2×2 square minus the area of 4 of a quarter of each of the large outer disks: $Area=2\times 2-4\left(\frac{1}{4}\pi(1)^2\right)=4-\pi$.
3. The darkest gray paint needs to cover the area of the one middle disk minus the area covered by the medium light gray paint: $Area=\pi(1)^2-(4-\pi)=2\pi-4$.

we need to divide the coins 1,2,3,4 into two groups such that one sums to 2 more than the other.

So only 7,3,1 and 5,4,2.

Remove the 4: The only possible way to get equal weights is 1+2+3+6= 5+7.

So the demonstration is to put the 6 coin aside, and put the 7,3,1 and 5,4,2